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EVOLUTION EQUATIONS AND FUNCTIONAL ANALYSIS GROUP

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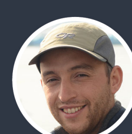

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Welcome

It is with great honor that we welcome you to the **XIV CONGRESS GAFEVOL 2021**
A conference in Honor of Carlos Lizama on the occasion of his 60th birthday.

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XIV CONGRESS GAFEVOL 2021

A conference in Honor of Carlos Lizama on the occasion of his 60th birthday

October 4–8, 2021 – virtual format

Contents

Abstracts of the Talks

Discrete Hölder spaces and their characterization via semigroups associated to the discrete Laplacian

Luciano Abadías Ullod*
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Universidad de Zaragoza
Zaragoza, Spain

Abstract

In this talk I will present the results obtained in [1]. We characterize the discrete Hölder spaces by means of the heat and Poisson semigroups associated to the discrete Laplacian. These characterizations allow us to get regularity properties of fractional powers of the discrete Laplacian and the Bessel potentials along these spaces and also in the discrete Zygmund spaces in a more direct way than using the pointwise definition of the spaces.

To obtain our results, it has been crucial to get boundedness properties of the heat and Poisson kernels and their derivatives in both space and time variables. We believe that these estimates are also of independent interest.

Joint work with:
Marta De León-Contreras[1], Department of Mathematical Sciences, University of Science and Technology, Trondheim, Norway.

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[1] Abadias, Luciano; De León-Contreras, Marta, *Discrete Hölder spaces, their characterization via semigroups associated to the discrete Laplacian and kernels estimates*, Send to publish (2021).

*Partially supported by Project PID2019-105979GB-I00 of the MICINN of Spain, Project E26-17R, D.G. Aragón, Universidad de Zaragoza, Spain, and Project for Young Researchers JIUZ-2019-CIE-01 of Fundación Ibercaja and Universidad de Zaragoza, Spain, e-mail: `labadias@unizar.es`

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Some results about abstract differential equations with state-dependent delay with non-instantaneous and without pre-defined impulses

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Abstract

Our goal is to extend the results obtained in [1] to abstract differential equations with state-dependent delay, given by:

$$u'(t) = Au(t) + f(t, u_{\eta(t,u_t)}), \; t \in [s_u^i, t_u^{i+1}], H(u(s_u^i)) = 0, \tag{1}$$

$$u(t) = \Psi(t_u^{i+1}, t, u(t_u^{i+1}), u(t)), \; t \in (t_u^{i+1}, s_u^{i+1}], H(u(t_u^{i+1})) = 1, \tag{2}$$

$$u_0 = \phi, \tag{3}$$

considering a impulsive action represented by the equation (2) that start at non-predefined times.

Joint work with:
Marta C. Gadotti[1], Departmento de Matemática, Universidade Estadual Paulista, Rio Claro, Brasil.

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Second and third order forward difference operator: What is in between?

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Universidad de Santiago de Chile
Santiago, Chile

Abstract

In this talk we present a new geometrical notion for a real-valued function defined in a discrete domain that depends on a parameter $\alpha \geq 2$. We give examples to illustrate connections between convexity and this new concept. We then prove two criteria based on the sign of the discrete fractional operator of a function u, $\Delta^\alpha u$ with $2 \leq \alpha < 4$. Two examples show that the given criteria are optimal with respect to the established geometrical notion. [1]

Joint work with:
Carlos Lizama[1], Departamento de Matemática y Ciencia de la Computación, Universidad de Santiago de Chile, Santiago, Chile.
Silvia Rueda[2], Departamento de Matemática y Ciencia de la Computación, Universidad de Santiago de Chile, Santiago, Chile.

References

[1] J. Bravo, C. Lizama, and S. Rueda. *Second and third order forward difference operator: what is in between?*. RACSAM **115**, 86 (2021). https://doi.org/10.1007/s13398-021-01015-5

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Non-local partial differential equations with delays and memory

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Abstract

In this talk we analyze a non-autonomous nonlocal functional parabolic equations when the external force contains hereditary characteristics involving bounded and unbounded delays. First, well-posedness of the problem is analyzed by the Galerkin method and energy estimations. Moreover, some results related to strong solutions are proved under suitable assumptions. The existence of stationary solutions is then established by a corollary of the Brower fixed point theorem. By constructing appropriate Lyapunov functionals in terms of the characteristic delay terms, a deep analysis on stability and attractiveness of the stationary solutions is established. ([1])

Joint work with:
Jiaohui Xu[1], Depto. Ecuaciones Diferenciales y Análisis Numérico
Universidad de Sevilla
Sevilla, Spain

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*Partially supported by grants PGC2018-096540-B-I00 and US-1254251 (Spain), e-mail: caraball@us.es

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A Way How Bloch Waves can understand Hashin-Shtrikman Microstructures

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Abstract

In this lecture we apply spectral methods by using the Bloch waves to study the homogenization process in the non-periodic class of generalized Hashin-Shtrikman micro-structures (Tartar in "The General Theory of Homogenization", volume 7 of Lecture Notes of the Unione Matematica Italiana, Springer, Berlin, p. 281, 2009), which incorporates both translation and dilation with a family of scales, including one subclass of laminates. We establish the classical homogenization result providing the spectral representation of the homogenized coefficients. It offers a new lead towards extending the Bloch spectral analysis in a non-periodic to non-commutative class of micro-structures.

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Quantification of fractional exponents in anomalous diffusion processes.

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Abstract

Particles in a fluid (liquid or gas) apparently present random movements caused by interactions with each other in the fluid. Brown initially observed this phenomenon in 1827, when studying the movement of particles trapped in pollen grains in water [Bro28]. However, it was not until almost 100 years later that Einstein conjectured that this type of displacement was due to the different ways in which particles collide with each other [SCRS89]. They move following a diffusion model whose solution is a normalized Gaussian distribution, whose variance increases linearly respect to the time. This fact was later verified by Perrin [Per09].

After more than a century, the classification of these movements is still an open problem. The *Mean Square Displacement (MSD)* of these movements follows an expression of the type t^α, where t is the time, and α is a positive number different from 1. If $\alpha < 1$, then we say that the process is *subdiffusive* and if $\alpha > 1$, we say that it is *superdiffusive*, linking both phenomena with fractional calculus. Among these processes, we find *random walks* (RW), *fractional Brownian movements* (FBM), and *Lévy's Walks* (LW).

With the recent advances in optical microscopy, Single Particle Tracking (SPT) appears to be a potential approach for studying different dynamic processes in life sciences [MGP15]. In particular, one can analyze the behavior of single molecules within living cells and tissues.

Recently, the Anomalous Diffusion Challenge was conducted in order to join different research groups for developing computational methods in order to infer the fractional exponent and to classify noisy trajectories generated using the aforementioned models [MnGVGM+20]. We report here some of our results in both tasks in order to show some connections between machine learning methods and fractional calculus problems [GBGMC], and we compare the results within the context of the challenge [MnGVGML+21].

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Joint work with:
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Alba Baeza Bosca[2], Instituto Universitario de Matemática Pura y Aplicada, Universitat Politècnica de València, València, Spain.
Miguel A. García-March[3], Instituto Universitario de Matemática Pura y Aplicada, Universitat Politècnica de València, València, Spain.

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[3] Partially supported by Spanish Ministry of Education and Vocational Training (MEFP) through the Beatriz Galindo program 2018 (BEAGAL18/00203) and Spanish Ministry MINECO (FIDEUA PID2019-106901GBI00/10.13039/501100011033), e-mail: albabos@posgrado.upv.es

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Poisson equation for (C, α)-bounded operators of fractional order

José E. Galé*
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University
City, Country

Abstract

Characterizations of fractional Poisson equations are given, together with a description of the associated one-sided Hilbert transform, for Cesàro bounded operators of fractional order. This generalizes well known results existing in the literature in the case $a = 0$.

Joint work with:
Luciano Abadías[1], Department, University, City, Country.
Carlos Lizama[2], Department, University, City, Country.

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On the stabilization of coupled system of two Korteweg-de Vries equations

Fernando A. Gallego Departamento de Matemática
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Manizales, Colombia

Abstract

In this talk, we present some recent results related to the rapid boundary stabilization for the Boussinesq System of the KdV-KdV Type on a Bounded interval introduced by J. Bona, M. Chen and J.-C. Saut. This is a model for the motion of small amplitude long waves on the surface of an ideal fluid. Here, we will consider the Boussinesq system of KdV-KdV type posed on a finite domain, with homogeneous Dirichlet- Neumann boundary controls acting at the right end point of the interval. Firstly, we build suitable integral transformations to get a feedback control law that leads to the stabilization of the system. More precisely, we will prove that the solution of the nonlinear closed-loop system decays exponentially to zero in the $L^2(0, L)$- norm and the decay rate can be tuned to be as large as desired if the initial data is small enough under the effects of two boundary feedback. Moreover, by using a Gramian-based method introduced by Urquiza to design our feedback control, we show that the solutions of the linearized system decay uniformly to zero when the feedback control is applied. The decay rate can be chosen as large as we want. The main novelty of our work is that we can exponentially stabilize this system of two coupled equations using only one scalar input.

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Monotonicity and Convexity for Discrete Fractional Operators by Using a Convolution Apporach

Christopher S. Goodrich*
School of Mathematics and Statistics
UNSW Sydney
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Abstract

This is a joint work with Prof. Carlos Lizama [1, 2]. I will discuss the connections between the sign of a discrete fractional sequential operator evaluated at a point n, such as

$$\Big((\Delta^{\beta} \circ \Delta^{\alpha})u\Big)(n),$$

and the monotonicity and convexity of the map u. Some representative results in the area will be discussed. In particular, I will demonstrate that considering the discrete fractional operator as a discrete convolution can be of great help in deducing these sorts of results, and that this approach improves and refines recent results in the area.

Joint work with:
Carlos Lizama[1], Departamento de Matemática y Ciencia de la Computación, Universidad de Santiago de Chile, Santiago, Chile.

References

[1] C. S. Goodrich, C. Lizama, *A transference principle for nonlocal operators using a convolutional approach: fractional monotonicity and convexity*, Israel J. Math. **236** (2020), 533–589.

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Nonlinear wave equations with an uneven bottom

Anna Karczewska*
Institute of Mathematics,
University of Zielona Góra
Zielona Góra, Poland

Abstract

We consider the model of an ideal fluid (incompressible and inviscid) moving irrotationally. We give a survey of derivations of Korteweg-de Vries-type equations with an uneven bottom for several cases when small (perturbation) parameters α, β, δ are of different orders [1]. Besides usual small parameters α and β, determining nonlinearity and dispersion, the model introduces the third parameter δ, which is related to bottom variations. Several different cases of the ordering of small parameters are discussed. In each case, one obtains the set of four Euler's equations containing the Laplace equation for the velocity potential, boundary condition at the bottom, and kinematic and dynamic boundary conditions at the unknown surface. Using perturbation theory and applying two first equations, one obtains the set of two coupled Boussinesq's equations for two unknown functions, the surface profile function $\eta(x,t)$ and the function $f(x,t)$ generating the velocity potential. Surprisingly, for all these cases, the resulting Boussinesq equations can be made compatible only for the particular piecewise linear bottom profiles, and the correction term in the final wave equations has a universal form. For such bottom relief, several new KdV-type wave equations are derived. These equations generalize the KdV, the extended KdV (KdV2), the fifth-order KdV (KdV5), and the Gardner equations. Examples of numerical simulations of the solutions to some of these equations are presented and discussed.

Joint work with:
Piotr Rozmej[1], Institute of Physics, University of Zielona Góra. Zielona Góra, Poland.

References

[1] Karczewska, Anna and Rozmej, Piotr, *Can simple KdV-type equations be derived for shallow water problem with bottom bathymetry?* Commun. Nonlinear Sci. Numer. Simulat. **82** (2020). 105073.

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Volterra-Stieltjes integral equations and semigroups on time scales

Jaqueline Godoy Mesquita*
Department of Mathematics
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Abstract

This lecture will divided in two parts. In the first part, we will discuss the Voltera-Stieltjes integral equations and their applications. This is a joint work with Edgardo Alvarez (Universidad del Norte, Colombia), Rogelio Grau (Universidad del Norte, Colombia) and Carlos Lizama (USACH, Chile).

The second part of my talk concerns about C_0-semigroup on time scales, which unifies the continuous, discrete and the cases which are between them. Also, it extends the classical theory of operator semigroups to the quantum case. We will investigate the relationship between the semigroup and its infinitesimal generator. We apply our theory to study the homogeneous and non homogeneous abstract Cauchy problem in Banach and Fréchet spaces. This is a joint work with Carlos Lizama (USACH, Chile) and HernÃ¡n HenrÃquez (USACH, Chile).

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Functions and semigroups, my overview in the last thirdteen years

Pedro J. Miana*

Departmento de Matemáticas

Universidad de Zaragoza

Zaragoza, Spain

Abstract

In this talk, we present a short overview of our collaboration with Carlos Lizama during the last thirdteen years. From [1] until [2] we give the hidden story of eight research papers and uncountable moments of Mathematics, travels, good food and beers.

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[1] Keyantuo, V.; Lizama,C.; Miana, P. J., *Defined Via Convoluted Semigroups and Cosine Functions*, Journal of Functional Analysis. **257**, (2009). 3454 - 3487.

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Nonlocal operators are chaotic

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València, Spain

Abstract

We study the dynamics of the fractional difference operator in the sense of Riemann-Liouville Δ^α for $0 < \alpha < 1$. We also characterize the chaotic behavior of nonlocal operators that come from a broad class of time stepping schemes of approximation for fractional differential operators. For that purpose, we use criteria for chaos of Toeplitz operators in Lebesgue spaces of sequences.

Joint work with:
Carlos Lizama[1], Departamento de Matemática y Ciencia de la Computación
Universidad de Santiago de Chile.
Alfred Peris[2], Institut Universitari de Matemàtica Pura i Aplicada
Universitat Politècnica de València, València, Spain

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[7] C. Lizama, M. Murillo-Arcila and A. Peris. Nonlocal operators are chaotic. Chaos, 30 (2020), 103126.

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Chaotic behaviour for certain numerical schemes

Alfred Peris*
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Abstract

We study the presence of chaos on dynamical systems associated to certain finite differences numerical schemes of a partial differential equation. Under some conditions, the dynamical system associated to a numerical scheme can be regarded as generated by a Toeplitz operator. The chaotic behaviour of Toeplitz operators was studied by Godefroy and Shapiro [4] and deLaubenfels and Emamirad [3] and, recently, by other researchers (see for instance [1, 5, 6, 7] and references therein).

We will present how to establish, under very general assumptions, the chaotic behaviour of these numerical schemes for PDEs, and illustrate them with some classical examples.

Joint work with:
Salud Bartoll[1], IUMPA, Universitat Politècnica de València
Félix Martínez-Giménez[2], IUMPA, Universitat Politècnica de València
Francisco Rodenas[3], IUMPA, Universitat Politècnica de València

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An evolution model describing solute trapping during solidification of binary alloys

Gabriela Planas*

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Abstract

In this talk, we present a mathematical analysis to a Cahn-Hilliard/Allen-Cahn system with degenerate mobility that models an isothermal process of solidification of a binary alloy. This model is able to predict an observable phenomenon called solute trapping. The existence of global weak solutions for the system is proved. We approximate the degenerate system and show the convergence of solutions to the approximated non-degenerate problem to a solution of the degenerate one. We also investigated deeply the non-degenerate system by showing the existence of global weak solutions, the existence of global strong solutions in the two-dimensional case, and local strong solutions in the three-dimensional case, as well as, providing conditions to the uniqueness be satisfied.

Joint work with:

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XIV CONGRESS GAFEVOL 2021
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A nonlocal in-time telegraph equation and some properties of its fundamental solution

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Abstract

In this talk, we consider a nonlocal in-time version of the so-called telegraph equation. We will show that the corresponding fundamental solution coincides with the probability density function of a stochastic process. Furthermore, using the theory of Volterra integral equations, we show that this type of processes is strongly related with the theory of sub-diffusive processes.

Joint work with:
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Integral Transforms in the Study of Nonlocal Operators

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Abstract

This talk is intended to present a series of results by myself and several collaborators, on the qualitative properties of the nonlocal operators. Our main motivation originates in the context of a certain class of equations, such as (linearized versions of) nonlocal field equations of motion appearing in nonlocal cosmology and other areas of theoretical physics.

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EXAMPLES ON FRACTIONAL OPEN QUANTUM DYNAMICS

ROLANDO REBOLLEDO

Resumen. The prolific contribution of Vassily Tarasov, among many, on quantum fractional dynamics, has enriched during this century the study of open quantum dynamics. He has thus opened up a fertile field of interdisciplinary work in Functional Analysis that allows the analysis of non-linear dynamics inspired by physical and biological models. There is an increasing list of references connected with this new research field (see eg.[9] [10],[2],[11][12] [13],[1],[4]). My talk aims to motivate work on these analytical methods using an example inspired by a well-known quantum dynamical semigroup (dissipative harmonic oscillator). I will show classical reductions of this semigroup related to the phenomenon known as quantum decoherence (see eg. [6],[7],[8], [3],[5]).

I dedicate these lines to my friend Carlos Lizama on his 60th birthday, with whom we have shared many mathematical concerns for a long time. I thank Carlos for his generous willingness to support and share projects that unite our respective lines of research.

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Approximation of solutions for state-dependent delay differential equations

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Abstract
Differential equations with state-dependent delays constitute a vast field of research, as much as in the theoretical part about the existence of solution and qualitative properties, as in the applications and techniques of solution. Concerning the techniques of solution, the numerical methods stand out, in particular those that generate continuous solutions. The objective of this speech is to remark the challenges that guide the researches on the numerical methods for functional differential equations, some results present in the literature and what is still to be done. To conclude, we present some approximation of solutions obtained with the computational program that is in development by the DEIAS research group from the Computational and Mathematics Department of FFCLRP/USP.

Joint work with:
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Nonlocal discrete equations

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Abstract

Let us consider the discrete Laplacian given by

$$-\Delta_d u(j) := -\big(u(j+1) - 2u(j) + u(j-1)\big), \quad j \in \mathbb{Z},$$

for $u : \mathbb{Z} \to \mathbb{R}$. The fractional powers of the discrete Laplacian $(-\Delta_d)^s$ on $\mathbb{Z}$, for $0 < s < 1$, can be defined with the approach of semigroups. In this talk I will present a theory of analytic semigroups and cosine operators generated by Δ_d and $-(-\Delta_d)^s$. Such a theory will be applied to prove existence and uniqueness of almost periodic solutions to equations of the form

$$\partial_t u(t,j) = Lu(t,j) + f(u(t,j),j); \partial_t u(t,j) = iLu(t,j) + f(u(t,j),j)$$

and

$$\partial_{tt} u(t,j) = Lu(t,j) + f(u(t,j),j),$$

where $j \in \mathbb{Z}$, $t \in (0,\infty)$, and L is taken to be either Δ_d or $-(-\Delta_d)^s$.

The results were obtained in collaboration with Carlos Lizama.

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Analytical properties of nonlocal discrete operators: Convexity

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Abstract

In this work, we prove new convexity results for nonlocal discrete operators across an entire region that covers sequential orders in two parameters. We review and extend current studies on the properties of positivity, monotonicity and convexity, explore borderline cases, and provide new insights on such properties by means of original examples evidencing the sharpness of the results. In passing, we provide a plausible answer to the converse of Dahal-Goodrich's monotonicity conjecture. Our method is based on the novel principle of transference.

Joint work with:
Jennifer Bravo[1], Departamento de Matemática y Ciencia de la Computación, Universidad de Santiago de Chile, Santiago, Chile.
Carlos Lizama[2], Departamento de Matemática y Ciencia de la Computación, Universidad de Santiago de Chile, Santiago, Chile.

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The peak-end rule and differential equations with maxima: a view on the unpredictability of happiness

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Abstract

In the 1990s, after a series of experiments, behavioural psychologist (and Nobel laureate in economics) Daniel Kahneman and his colleagues formulated the following Peak-End evaluation rule: *the remembered utility of pleasant or unpleasant episodes is accurately predicted by averaging the Peak (most intense value) of instant utility (or disutility) recorded during an episode and the instant utility recorded near the end of the experience* [**?**, p. 381]. Hence, the simplest mathematical model for time evolution of the experienced utility function $u = u(t)$ can be given by the scalar differential equation $u'(t) = au(t) + b \max_{s\in[t-h,t]} u(s) + f(t)$ $(*)$, where f represents exogenous stimuli, h is the maximal duration of experience and $a, b \in R$ are some averaging weights. We present an approach allowing, for a range of parameters a, b, h and periodic sine-like term f, to describe the dynamics of $(*)$ in terms of some associated one-dimensional dynamical system generated by piece-wise continuous maps of a finite interval into itself. Then we prove that the hedonic utility $u(t)$ ('happiness') can exhibit chaotic behavior. [**?**]

Joint work with:
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Elena Trofimchuk: Department of Differential Equations, Igor Sikorsky Kyiv Polytechnic Institute, Kyiv, Ukraine.

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Elliptic systems involving Schrödinger operators with vanishing potentials

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Abstract

We prove the existence of a bounded positive solution of the following elliptic system involving Schrödinger operators

$$\begin{cases} -\Delta u + V_1(x)u = \lambda\rho_1(x)(u+1)^r(v+1)^p & \text{in} \quad \mathbb{R}^N, \\ -\Delta v + V_2(x)v = \mu\rho_2(x)(u+1)^q(v+1)^w & \text{in} \quad \mathbb{R}^N, \\ u(x)v(x) \to 0 & \text{as} \quad |x| \to \infty \end{cases}$$

where $p, q, r, s \geq 0$, V_i is a nonnegative vanishing potential, and ρ_i has the property (H) introduced by Brezis and Kamin [1]. As in that celebrated work we will prove that for every $R > 0$ there is a solution (u_R, v_R) defined on the ball of radius R centered at the origin. Then, we will show that this sequence of solutions tends to a bounded solution of the previous system when R tends to infinity. Furthermore, by imposing some restrictions on the powers p, q, r, s without additional hypotheses on the weights ρ_i, we obtain a second solution using variational methods. In this context we consider two particular cases: a gradient system and a Hamiltonian system.

Joint work with:
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Exponential turnpike property for fractional control problems

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Abstract

In this talk we consider averages convergence as the time-horizon goes to infinity of optimal solutions of time-dependent control problems to optimal solutions of the corresponding stationary optimal control problems. Control problems play a key role in engineering, economics, and sciences. To be more precise, in climate sciences, often times, relevant problems are formulated in long time scales, so that, the problem of possible asymptotic behaviors when the time-horizon goes to infinity becomes natural. Assuming that the controlled dynamics under consideration are stabilizable towards a stationary solution, the following natural question arises: Do time averages of optimal controls and trajectories converge to the stationary optimal controls and states as the time-horizon goes to infinity? This question is very closely related to the so-called turnpike property that shows that, often times, the optimal trajectory joining two points that are far apart, consists in, departing from the point of origin, rapidly getting close to the steady-state (the turnpike) to stay there most of the time, to quit it only very close to the final destination and time. In the present talk we are dealing with control problems of fractional parabolic equations with non-zero exterior data (Dirichlet and non-local Robin) associated with the fractional Laplace operator. We prove the turnpike property for the non-local Robin control problem and the exponential turnpike property for both Dirichlet and non-local Robin control problems.

Joint work with:
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